Name————————————, ID ——————

Exam I, MTH 320: Abstract Algebra I, Spring 2012

Ayman Badawi

QUESTION 1. a) Let $G = (Z, +)$. We know that $H =< 6 > \cap < 16 >$ is a cyclic subgroup of G. Find all generators of H.

b) Let $F = Z_5 \bigoplus U(8)$. Is F cyclic? if yes, then how many generators does F have? If no, then explain.

c) Let $F = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} \in M_2(Z_8)$. If possible find F^{-1} (i.e. is there a matrix F^{-1} such that $FF^{-1} = I_2$.) If not possible, then explain

d) Let $F = \begin{bmatrix} 3 & 5 \\ 1 & 5 \end{bmatrix} \in M_2(Z_8)$. If possible find F^{-1}. If not possible, then explain

e) Given that G is a group with 16 elements. Suppose there exists an element $a \in G$ such that $a^8 \neq e$. Prove that G is an abelian group.

f) Let $F =< a >$ be a cyclic group of order 24 and let $H < a^9 >$ be a subgroup of G. Find $|H|$. We know that H is cyclic, and hence find all generators of H.

h) Let G be a group with 27 elements. Given that G has exactly 2 elements of order 3, exactly 6 elements of order 9. Prove that G is an abelian group.

QUESTION 2. Let G be an abelian group. Given $a, b \in G$ such that $|a| = 20$ and $|b| = 42$. Find $|a^4 * b^2|$. Explain your answer.

a) Let $G = < a >$ be a cyclic group of order 12 and let H be a subgroup of G of order 4. Find all distinct left cosets of H.

b) Find the elements of A_3, then find all distinct left cosets of A_3.

c) Let $M = (2\ \ 3\ \ 5)\ o\ (2\ \ 6\ \ 5\ \ 1)$. Find $|M|$.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Exam II, MTH 320: Abstract Algebra I, Spring 2012

Ayman Badawi

QUESTION 1. a) We know that A_5 is a simple group. However, prove that A_5 does not have a normal subgroup of order 12.

b) Prove that A_5 does not have a cyclic subgroup of order 6.

QUESTION 2. (You may use the fact I proved in the class: Every abelian group of order P^2 (P is prime) is either Z_{P^2} or $Z_P \oplus Z_P$). Let G be an abelian group such that $|G| = P^3$ (P is prime). Given G has a cyclic subgroup of order P^2 but no element in G is of order P^3. Prove that G is isomorphic to $Z_P \oplus Z_{P^2}$.

b) Given G is a group such that $|G| = mp^n$, where p is prime, n, m positive integers, and $gcd(p, m) = 1$. Given H is a normal subgroup of G such that $|H| = p^n$. Assume that F is a subgroup of G such that $|F| = p^k$ for some positive integer $k \leq n$. Prove that $F \subseteq H$.

c) Given G is an abelian group such that $|G| = mp^n$, where p is prime, n, m positive integers, and $gcd(p, m) = 1$. Given H is s subgroup of G such that $|H| = p^n$. Assume that F is a subgroup of G such that $|F| = p^n$. Prove that $H = F$.

QUESTION 3. a) Prove that $U(8)$ is isomorphic to $Z_2 \oplus Z_2$

b) Is $U(32)$ isomorphic to Z_{16}? Explain briefly

c) Given $U(4) \oplus U(9)$ is isomorphic to $Z_a \oplus Z_b \oplus Z_c \oplus Z_d$. Find a, b, c, d; note that a, b, c, d are positive integers and each is bigger than one. (If you need to explain, then make it very brief).

d) Construct a group homomorphism say $F: Z_{12} \Rightarrow Z_3$ such that F is not the trivial group homomorphism. Find $Range(F)$ and $Ker(F)$.

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E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name——————————————, **ID** ——————

MTH 320, ABSTRACT ALGEBRA , Final Exam

Ayman Badawi

QUESTION 1. Let F be a non-cyclic abelian group with 126 elements. Up to group isomorphism find all such groups. Explain Briefly.

QUESTION 2. Let G be a group with 33 elements such that G has a normal subgroup H with 11 elements. Suppose there is an element $b \in G$ and $b \notin H$ such that $|b| \neq 3$. Prove that G is an abelien group. Up to isomorphism, find all such groups.

QUESTION 3. Let D be a group with 70 elements. Given there are two elements say a and b in D such that $a * b = b * a$, $|a| = 7$, and $|b| = 5$. Prove that D has exactly one subgroup of order 35.

QUESTION 4. Construct a group homomorphism say F from $S_3 \oplus Z_5$ into Z_{30} such that $|Image(F)| = 5$. Find $Ker(F)$.

QUESTION 5. a) Let $F = Z_6 \oplus Z_8$.

(i) Is there an element in F of order 48? if yes, then find it. If no, then explain

(ii) Is there an element in F of order 24? if yes, then find it. If no, then explain

(iii) If possible, construct a cyclic subgroup of F with 12 elements.

(iv) If possible, construct a non-cyclic subgroup of F with 12 elements.

QUESTION 6. b) Let $F = (1\ 4\ 5\ 3)\ 0\ (2\ 1\ 3\ 4) \in S_5$. Find $|F|$.

c) Is $(13)\ 0\ (3\ 2\ 1)\ 0\ (1\ 7\ 4) \in A_7$? Explain

QUESTION 7. Give me an example a group D such that D is not abelian, but D has a cyclic normal subgroup H where D/H is also cyclic .

QUESTION 8. Let $F = U(4)4 \oplus U(8)$. Then $H = \{(1, 1), (1, 3)\}$ is a subgroup of F. Find all distinct left cosets of H

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

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MTH 320, ABSTRACT ALGEBRA , Final Exam

Ayman Badawi

QUESTION 1. Let F be a group with 63 elements. Given $a \in F$, $a \neq e$, and $a^{38} = a^{10}$.
1) Find $|a|$.

2) Find $|a^3|$

QUESTION 2. Let D be an abelian group with 99 elements. Prove that D has exactly one subgroup of order 9.

QUESTION 3. Let D be an abelian group with 40 elements such that D has exactly two subgroups of order 10 . (Up to isomorphism) find the group D.

QUESTION 4. Is there a group homomorphism say V from $U(16)$ into Z_{16} such that $Image(V) = \{0, 2, 4, 6, 8, 10, 12, 14\}$? If yes, construct V. If no, explain clearly.

QUESTION 5. Let H be a group with 30 elements. Given $a, b \in H$ such that $|a| = 3$, $|b| = 5$ and $ab = ba$. Prove that H has exactly one subgroup of order 15.

QUESTION 6. a) Let $F = Z_4 \oplus Z_9$. Is there an element in F of order 6? if yes find it.

b) Let $F = (1\ 4\ 5)\ 0\ (2\ 1\ 3\ 4) \in S_5$. Find $|F|$.

c) Is $(13)\ 0\ (32)\ 0\ (1\ 7\ 4) \in A_7$? Explain

QUESTION 7. Give me an example of a group D such that D is not cyclic, but D has a cyclic normal subgroup H where D/H is also cyclic .

QUESTION 8. Let $F = Z_2 \oplus U(8)$. Then $H = \{(0,0),(1,7)\}$ is a subgroup of F. Find all distinct left cosets of H

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